Universität
Stuttgart

Weak and Classical Solutions for a Model Problem in Radiation Hydrodynamics

Rohde, C., Tiemann, N., Wen-An Yong

Berichte aus dem Institut für Angewandte Analysis und Numerische Simulation

Preprint 2006/013

Universität Stuttgart

Weak and Classical Solutions for a Model Problem in Radiation Hydrodynamics

Rohde, C., Tiemann, N., Wen-An Yong

Berichte aus dem Institut für Angewandte Analysis und Numerische Simulation

Preprint 2006/013

Institut für Angewandte Analysis und Numerische Simulation (IANS)
Fakultät Mathematik und Physik
Fachbereich Mathematik
Pfaffenwaldring 57
D-70 569 Stuttgart

E-Mail: ians-preprints@mathematik.uni-stuttgart.de
WWW: http://preprints.ians.uni-stuttgart.de

ISSN **1611-4176**

Weak and Classical Solutions for a Model Problem in Radiation Hydrodynamics

Christian Rohde[1], Nils Tiemann[2], and Wen-An Yong[3]

[1] Institut für Angewandte Analysis und Numerische Simulation, Universität Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany, crohde@mathematik.uni-stuttgart.de
[2] Fakultät für Mathematik Universität Bielefeld, Postfach 100 131, Germany, nils.tiemann@uni-bielefeld.de
[3] Zhou Pei-Yuan Center for Appl. Math., Tsinghua University, Beijing 100084, China, wayong@mail.tsinghua.edu.cn

Summary. It has been observed for a long time that radiation effects can prevent the development of singularities of shock-wave type in solutions for mathematical models for compressible flows. We consider a multi-dimensional model problem in the form of a system of nonlinear hyperbolic balance laws and prove that the associated Cauchy problem can have smooth global solutions provided that the initial data is sufficiently close to an equilibrium state. Numerical experiments confirm this result but also show that shock-waves can develop for large amplitude initial data.

1 Introduction

A widely used mathematical model for radiation-driven ideal compressible flows, in particular in astrophysics [4, 5], is given in $\mathbb{R}^d \times (0,T)$, $T>0$, $d \in \{1,2,3\}$, by the equations of gas dynamics coupled via an integral-type source term to a family of radiation transport equations:

$$\begin{aligned} \rho_t + \operatorname{div}(\rho v) &= 0, \\ (\rho v)_t + \operatorname{div}(\rho v \otimes v + p\, Id) &= 0, \\ (\rho e)_t + \operatorname{div}(\rho e v + p v) &= \rho \oint_{\mathcal{S}^{d-1}} \kappa(I(.,\omega) - B(\theta)) d\omega, \\ I(.,\omega)_t + c\omega \cdot \nabla I(.,\omega) &= c\rho(B(\theta) - I(.,\omega)). \end{aligned} \tag{1}$$

Here the unknowns are the density of the fluid $\rho = \rho(x,t) > 0$, the velocity $v = v(x,t) \in \mathbb{R}^d$, the temperature $\theta = \theta(x,t) > 0$, and the radiation intensity $I = I(x,t,\omega) \geq 0$ for $(x,t) \in \mathbb{R}^d \times [0,\infty)$ and $\omega \in \mathcal{S}^{d-1}$. The vector ω denotes the direction of radiation. System (1) is closed by the relations

$$p = p(\rho,\theta), \quad B = B(\theta), \quad e = \epsilon(\rho,\theta) + \frac{1}{2}|v|^2,$$

where p denotes the given pressure function, $\kappa > 0$ a positive constant, B the Planck function and ε the specific energy, respectively. $c > 0$ is the speed of light. Under standard thermodynamical assumptions (1) turns out to be an infinite-dimensional hyperbolic balance law. We have to face the problem that solutions of (1) contain singularities of shock-wave type even if we consider the Cauchy problem for (1) with smooth initial data. However it has been observed early (see e.g. [1]) that the Cauchy problem for (1) might possess global smooth solutions due to radiative damping. In this contribution we will focus on the discrete-ordinate approximation for (1). The most basic approximation with two directions has been analyzed for the case of one space dimension in the seminal work of Kawashima and co-workers ([2] and references therein, see also the recent work [3] on profiles). Rigorous results for the general discrete-ordinate approximation can be found in [9]. However, up to our knowledge no results on global smooth solutions in multiple space dimensions are available.
With $\mathbf{f} := (f^1, \dots, f^d)^T \in C^\infty(\mathbb{R}_{>0}, \mathbb{R}^d)$ a multi-dimensional model problem for (1) in $\mathbb{R}^d \times (0, \infty)$ and for $\omega \in \mathcal{S}^{d-1}$ is given by

$$\begin{aligned} u_t + f^1(u)_{x_1} + \dots + f^d(u)_{x_d} &= \kappa \oint_{\mathcal{S}^{d-1}} (I(.,\omega) - B(u))\, d\omega, \\ I(.,\omega)_t + c\omega \cdot \nabla I(.,\omega) &= c\rho(B(u) - I(.,\omega)). \end{aligned} \tag{2}$$

This model relates to (1) as a scalar nonlinear conservation law to the gas dynamics equations. The unknown u takes the role of a lumped quantity of density, velocity and temperature. Since (2) is still a (more difficult to handle) infinite-dimensional system we introduce a finite-dimensional approximation. For this sake let for each $L \in \mathbb{N}$ a partition of the unit sphere $\mathcal{S}^{d-1}$ into L subsets $\Omega_1, \Omega_2, \dots, \Omega_L$ of equal size be given such that it satisfies properties

$$\mathcal{S}^{d-1} = \Omega_1 \cup \dots \cup \Omega_L,\ \mathring{\Omega}_l \cap \mathring{\Omega}_k = \emptyset\ \forall\ \ k \neq l. \tag{3}$$

Choose for $l = 1, \dots, L$ the discrete ordinates $\omega^l \in \Omega_l$ as arbitrary vectors and define

$$\sigma_L := |\Omega_l| = L^{-1}|\mathcal{S}^{d-1}|. \tag{4}$$

Now, consider for the unknown $U = (u, I^1, \dots, I^L)^T : \mathbb{R}^d \times [0, \infty) \to \mathcal{U} := \mathbb{R}^{L+1}_{>0}$ the Cauchy problem

$$\begin{aligned} u_t + f^1(u)_{x_1} + \dots + f^d(u)_{x_d} &= \kappa\sigma_L \sum_{l=1}^{L} (I_l - B(u)) \\ I^1_t + c\omega^1 \cdot \nabla I^1 &= c(B(u) - I^1) \qquad \text{in } \mathbb{R}^d \times (0,\infty) \\ &\dots \\ I^L_t + c\omega^L \cdot \nabla I^L &= c(B(u) - I^L) \end{aligned} \tag{5}$$

with

$$u(.,0) = u_0,\ I^1(.,0) = I^1_0, \dots, I^L(.,0) = I^L_0 \text{ in } \mathbb{R}^d. \tag{6}$$

For the Planck function $B \in C^\infty(\mathbb{R}_{>0})$ we only assume

$$B(u) > 0,\ B'(u) > 0 \qquad (u \in \mathbb{R}_{>0}). \tag{7}$$

With obvious definitions for $F_1, \dots, F_d, Q : \mathcal{U} \to \mathbb{R}^{L+1}$ we write (5) in the compact form

$$U_t + F_1(U)_{x_1} + \cdots + F_d(U)_{x_d} = Q(U). \tag{8}$$

Let us give a short outline of the paper. In Sect. 2 we review an existence result for global weak entropy solutions for (5),(6) in the framework of BV-techniques taken from [8]. In Sect. 3 the main result of this paper, namely Theorem 3 is presented and proven.

In the concluding Sect. 4 we display several numerical experiments which show more details about the mechanism of radiative smoothing and confirm our analytical findings.

2 The Existence of Global Weak Solutions

A weak solution of (5), (6) is a function $(u, I^1, \ldots, I^L) \in L^\infty_{loc}(\mathbb{R}^d \times [0,\infty))^{L+1}$ such that

$$\begin{aligned}\int_{\mathbb{R}^d}\int_0^\infty & u\varphi_t + \mathbf{f}(u)\cdot\nabla\varphi\, dxdt \\ &= -\int_{\mathbb{R}^d} u_0\varphi(.,0)\,dx - \int_{\mathbb{R}^d}\int_0^\infty \varphi\kappa\sigma_L \sum_{l=1}^L \big(I^l - B(u)\big)\,dxdt,\end{aligned}$$

$$\int_{\mathbb{R}^d}\int_0^\infty I^l\varphi_t + cI^l\omega^l\cdot\nabla\varphi\,dxdt = -\int_{\mathbb{R}^d} I_0^l\varphi(.,0)\,dx - c\int_{\mathbb{R}^d}\int_0^\infty \big(B(u) - I^l\big)\varphi\,dxdt$$

holds for $l = 1, \ldots, L$ and all $\varphi \in C_0^\infty(\mathbb{R}^d \times [0,\infty))$.

Theorem 1. *Let $u_0, I_0^1, \ldots, I_0^L \in L^\infty(\mathbb{R}^d) \cap BV(\mathbb{R}^d)$ and $b > a > 0$ with*

$$(u_0(x), I_0^1(x), \ldots, I_0^L(x)) \in [a,b] \times [B(a), B(b)]^L \quad \textit{for almost all } x.$$

Then there is a weak solution $(u, I^1, \ldots, I^L)$ of the Cauchy problem (5),(6) that satisfies for almost all (x,t)

$$\begin{aligned}(u(x,t), I^1(x,t), \ldots, I^L(x,t)) &\in [a,b]\times[B(a),B(b)]^L, \\ |u(.,t)|_{BV(\mathbb{R}^d)} + \frac{\kappa\sigma_L}{c}\sum_{l=1}^L |I^l(.,t)|_{BV(\mathbb{R}^d)} &\le |u_0|_{BV(\mathbb{R}^d)} + \frac{\kappa\sigma_L}{c}\sum_{l=1}^L |I_0^l|_{BV(\mathbb{R}^d)}.\end{aligned} \tag{9}$$

The proof of this theorem via a numerical approximation can be found in [8]. We remark in passing that the estimates on u in (9) are uniformly with respect to $c \to \infty$. This allows to perform this non-relativistic limit. In [8] it is shown that the u-component of the limit satisfies a hyperbolic integro-differential equation.

3 The Existence of Global Classical Solutions

In this section we analyze the regularity of the weak solutions for (5), (6) which exist due to the result in Sect. 2. We show that the Cauchy problem (5), (6) admits global classical solutions provided the initial data is close to an equilibrium state in an appropriate Sobolev norm.

To obtain this result we shall apply a more general theory on smooth solutions for multi-dimensional hyperbolic balance laws by Yong [7]. More precisely, consider the Cauchy problem for a system of balance laws of the form

$$\begin{aligned} \hat{U}_t + \hat{F}_1(\hat{U})_{x_1} + \cdots + \hat{F}_d(\hat{U})_{x_d} = \hat{Q}(\hat{U}) := \begin{pmatrix} 0 \\ \hat{q}(\hat{u},\hat{v}) \end{pmatrix} \text{ in } \mathbb{R}^d \times (0,\infty), \\ \hat{U}(.,0) = \hat{U}_0 \text{ in } \mathbb{R}^d. \end{aligned} \tag{10}$$

Here $\hat{U} = (\hat{u}^T, \hat{v}^T)^T : \mathbb{R}^d \times [0,\infty) \to \hat{\mathcal{U}} \subset \mathbb{R}^n$, $n \in \mathbb{N}$, is the unknown split up into the components $\hat{u} : \mathbb{R} \times [0,\infty) \to \mathbb{R}^{n-r}$ and $\hat{v} : \mathbb{R} \times [0,\infty) \to \mathbb{R}^r$. $\hat{F}_1, \ldots, F_d, \hat{Q} \in C^\infty(\mathcal{U}, \mathbb{R}^n)$ are the flux and source functions where we assume that the first $n-r$ components of $\hat{Q}$ vanish and the last r components are given by the function $\hat{q} : \hat{\mathcal{U}} \to \mathbb{R}^r$. $\hat{U}_0 : \mathbb{R} \to \hat{\mathcal{U}}$ is the initial function.

Theorem 2 (from [7]). *Let $\hat{U}_e \in \hat{\mathcal{U}}$ be such that $\hat{Q}(\hat{U}_e) = 0$. We assume that we have*

(i) the Jacobian $D_{\hat{v}}\hat{q}(\hat{U}_e)$ is regular,
(ii) there is a strictly convex function $\hat{\eta} \in C^\infty(\hat{\mathcal{U}}, \mathbb{R})$ and functions $\Psi_1, \ldots, \Psi_d \in C^\infty(\hat{\mathcal{U}}, \mathbb{R})$ such that

$$\nabla_{\hat{U}}\hat{\eta}(\hat{U})^T D_{\hat{U}}\hat{F}_i(\hat{U}) = \nabla_{\hat{U}}\hat{\Psi}_i(\hat{U})^T \quad (\hat{U} \in G,\ i = 1, \ldots, n), \tag{11}$$

(iii) for all $G \subset\subset \hat{\mathcal{U}}$ there is a constant $c_G > 0$ such that

$$\Big(\nabla_{\hat{U}}\hat{\eta}(\hat{U}) - \nabla_{\hat{U}}\hat{\eta}(\hat{U}_e)\Big) \cdot \hat{Q}(\hat{U}) \le -c_G|\hat{Q}(\hat{U})|^2 \quad (\hat{U} \in G),$$

(iv) the kernel of the Jacobian $D_{\hat{U}}\hat{Q}(\hat{U}_e)$ contains no eigenvector of the matrix $n_1 D_{\hat{U}}\hat{F}_1(\hat{U}_e) + \cdots + n_d D_{\hat{U}}\hat{F}_d(\hat{U}_e)$ for any $n \in \mathcal{S}^{d-1}$.

Then, for $s \ge [d/2] + 2$, there exists a constant $C > 0$ such that for all initial functions $\hat{U}_0$ satisfying

$$\|\hat{U}_e - \hat{U}_0\|_{H^s(\mathbb{R}^d)} \le C$$

there is a unique classical solution $\hat{U} - \hat{U}_e \in C([0,\infty); H^s(\mathbb{R}^d))$ of (10).

To apply Theorem 2 to our radiation model problem (5) we reformulate (5) in the new variables

$$\hat{U} = (\hat{u}, \hat{v}_1, \ldots, \hat{v}_d) := \Big(u + \frac{\kappa\sigma_L}{c}\sum_{l=1}^{L} I^l, I^1, \ldots, I^L\Big)^T \in \hat{\mathcal{U}}. \tag{12}$$

We obtain then a system in the form (10) which is equivalent to (5), resp. (8), by the choices

$$\hat{F}_1(\hat{U}) := AF_1(A^{-1}\hat{U}), \ldots, \hat{F}_d(\hat{U}) := AF_d(A^{-1}\hat{U}),\ \hat{Q}(\hat{U}) := AQ(A^{-1}\hat{U}), \tag{13}$$

where $A \in \mathbb{R}^{(L+1)\times(L+1)}$ is the matrix with the first row $(1, (\kappa\sigma_L)/c, \cdots, (\kappa\sigma_L)/c)$ and the ith row vanishes except for the entry 1 at position i, $i = 2, \ldots, L+1$. Note that we have $r = L$ and

$$\hat{q}_i(\hat{u}, \hat{v}) = c\left(\hat{v}_i - B\Big(\hat{u} - \frac{\kappa\sigma_L}{c}\sum_{l=1}^{L}\hat{v}_l\Big)\right) \quad (i = 1, \ldots, L). \tag{14}$$

The main result of this section is

Theorem 3. *Let $L > 1$ and $s \geq [d/2] + 2$. Consider an equilibrium point $U_e := (u_e, I_e, \dots, I_e)^T \in \mathcal{U}$ of Q, i.e., with $B(u_e) = I_e$.*
Then there exists a constant $C > 0$ such that for all initial data with

$$\|U_0 - U_e\|_{H^s(\mathbb{R}^d)} \leq C$$

there is a classical solution $U - U_e \in C([0,\infty); H^s(\mathbb{R}^d))$ of (5), (6).

Proof We have to verify the conditions (i)-(iv) from Theorem 2. So, consider system (10) with the choices (12),(13). For (i) a straightforward computation for (14) leads to

$$\det\big(D_{\hat{v}}\hat{q}(\hat{u},\hat{v})\big) = c^L\Big(1 + L\frac{\kappa\sigma_L}{c}B'\Big(\hat{u} - \frac{\kappa\sigma_L}{c}\sum_{l=1}^{L}\hat{v}_l\Big)\Big) \qquad ((\hat{u},\hat{v}^T)^T \in \hat{\mathcal{U}}).$$

We conclude that $\det\big(D_{\hat{v}}\hat{q}(\hat{U}_e)\big) \neq 0$ with

$$\hat{U}_e := \big(u_e + \frac{|\mathcal{S}^{d-1}|}{c}B(u_e), B(u_e), \dots, B(u_e)\big)^T$$

holds since we have $B' \geq 0$ due to (7).
Turning to (ii) we observe: if $(\eta, \psi_1, \dots, \psi_d)$ is an entropy tuple for (5) then $(\hat{\eta}, \hat{\psi}_1, \dots, \hat{\psi}_d)$ with

$$\hat{\eta}(\hat{U}) = \eta(A^{-1}\hat{U}), \quad \hat{\psi}_i(\hat{U}) = \psi_i(A^{-1}\hat{U}) \quad (\hat{U} \in \hat{\mathcal{U}}), \tag{15}$$

is an entropy tuple for (10), i.e., $\hat{\eta}$ is convex and (11) holds. Therefore it suffices to construct an entropy tuple for (5) which we choose as

$$\begin{aligned} \eta(U) &= -\ln(u) + \frac{\kappa\sigma_L}{c}\sum_{l=1}^{L}\pi(I^l), \\ \psi_i(U) &= -\int_k^u \frac{f_i'(w)}{w}\,dw + \kappa\sigma_L\sum_{l=1}^{L}\omega_i^l\pi(I^l),\ k > 0. \end{aligned} \tag{16}$$

The function $\pi \in C^\infty(\mathbb{R}_{>0}, \mathbb{R})$ is given such that we have for $I > 0$

$$\pi'(I) = -\frac{1}{B^{-1}(I)}. \tag{17}$$

Obviously (11) is satisfied. The convexity of η follows due to (7) from

$$\pi''(I) = \frac{B^{-1\prime}(I)}{\big(B^{-1}(I)\big)^2} > 0 \quad (I > 0).$$

Thus we have verified (ii) in Theorem 2.
Since we can add any linear function to an entropy and still obtain an entropy we can assume w.l.o.g. $\nabla\eta(U_e) = 0$. Furthermore we have $\nabla\eta(U)\cdot Q(U) = \nabla_{\hat{U}}\hat{\eta}(\hat{U})\cdot\hat{Q}(\hat{U})$ for all $U \in G$, G any compact subset of $\mathcal{U}$, and $\hat{U} = A^{-1}U$. Therefore it is enough to check for (iii) that there is a constant $c_G > 0$ such that the estimate

$$\nabla_U\eta(U)\cdot Q(U) \leq -c_G|Q(U)|^2 \tag{18}$$

holds. We compute with (16) and (17)

$$\begin{aligned}\nabla_U \eta(U)\cdot Q(U) &= \kappa\sigma_L \sum_{l=1}^{L}(I^l - B(u))\frac{u - B^{-1}(I^l)}{uB^{-1}(I^l)}\\ &= -\kappa\sigma_L \sum_{l=1}^{L}\frac{(B(u)-I^l)^2}{uB^{-1}(I^l)}\frac{B^{-1}(B(u)) - B^{-1}(I^l)}{B(u)-I^l}\\ &\le -c_G|Q(U)|^2.\end{aligned}$$

The last inequality follows for some $c_G > 0$ from (7), $u > 0$, the definition of Q, and the compactness of the set G.
Finally we have to consider condition (iv). Again it suffices to show that the kernel of $D_U Q(U_e)$ contains no eigenvector of $n_1 D_U F_1(U_e) + \cdots + n_d D_U F_d(U_e)$ for no $n \in \mathcal{S}^{d-1}$. We compute

$$D_U Q(U) = \begin{pmatrix} -\kappa\sigma_L L B'(u_e) & \kappa\sigma_L & \cdots & \kappa\sigma_L \\ cB'(u_e) & -c & \cdots & 0 \\ \vdots & & \ddots & \\ cB'(u_e) & 0 & \cdots & -c \end{pmatrix}.$$

and thus the kernel of $D_U Q(U_e)$ is given by $\{\alpha(1, B'(u_e), \ldots, B'(u_e))^T \,|\, \alpha \in \mathbb{R}\}$. A short calculation shows that an element of this set can only be an eigenvector provided

$$cn\cdot\omega^l = n\cdot(f_1'(u_e), \ldots, f_d'(u_e))^T$$

holds for $l = 1, \ldots, L$. Due to $L > 1$ and (3) this cannot be true and we have verified (iv). □

4 Numerical Experiments for the Model Problem

Theorem 3 gives sufficient conditions for the existence of smooth global solutions. We underline this finding by a series of numerical experiments. We consider the model (5) for $d = 2$ in the domain $(0,1)^2$ and choose the (discontinuous) initial data

$$u_0(x) = \begin{cases} 1.5 & : \ |x - (0.5, 0.5)| \le 0.125 \\ 1 & : \ \text{elsewhere} \end{cases}, \qquad I_0^l(x) = 1.0 \quad (l = 1, \ldots, L).$$

As boundary conditions we assume periodic ones. For c we choose $c = 10$, the fluxes and B are given by

$$f^1(u) = f^2(u) = \frac{u^2}{2}, \quad B(u) = u^4.$$

The initial/boundary value problem is solved by a frist-order standard finite volume scheme on a uniform Cartesian mesh with mesh width 0.025. As numerical flux we use the Engquist-Osher flux in each component.
In Fig. 1 we display a contour plot of u for $\kappa = 0$ and $\kappa = 0.25$ computed with 8 equidistributed ordinates.

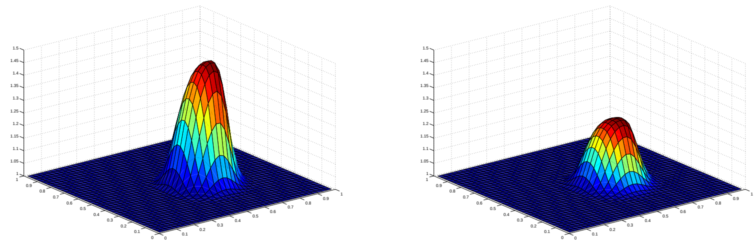

Fig. 1. Computational results for u with $\kappa = 0$ (left) and $\kappa = 8$, $L = 8$ (right) at time $t = 0.0625$.

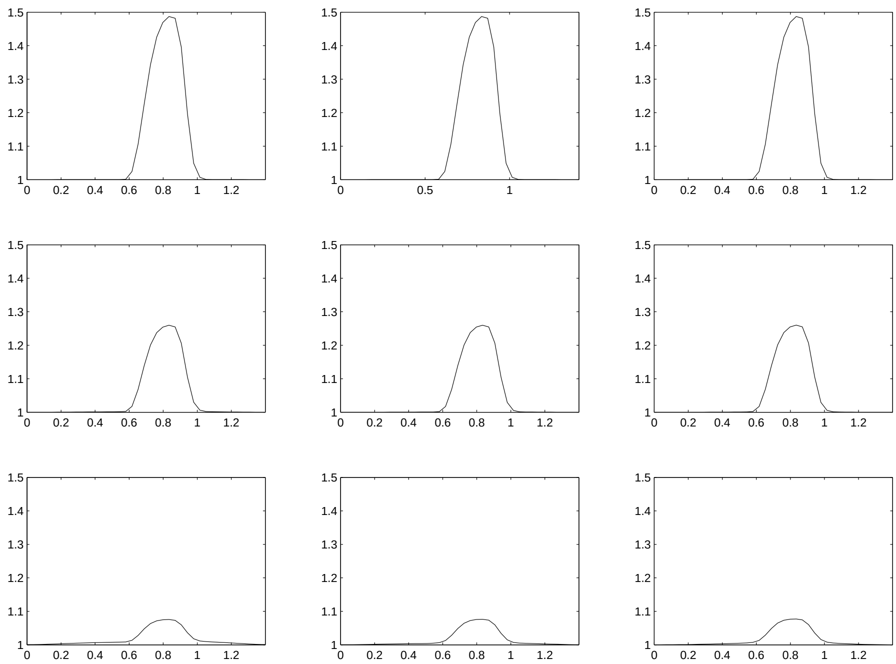

Fig. 2. Computational results for u restricted to the diagonal with $L = 4, 8, 16$ ordinates (from left to right) and $\kappa = 0.25, 0.5, 1.0$ (from top to bottom) at time $t = 0.0625$.

One observes clearly the strong damping effect of the radiation. More results are displayed in Fig. 2 where we display u restricted to the diagonal line connecting the coordinate points $(0, 1)$ with $(1, 0)$. We observe that with increasing the value of κ, that means to increase the radiative damping effect, the solution component

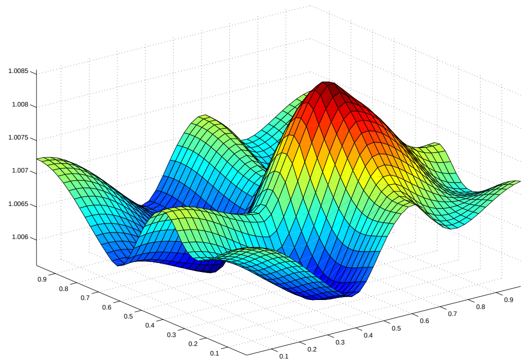

Fig. 3. Computational results for u with $\kappa = 1$ and $L = 16$ at time $t = 0.2$.

u itselfs decays and the initial discontinuity seems to be smeared out. Almost not visible in Fig. 2 is the difference which results from the choice of different numbers of ordinates. Finally we present in Fig. 3 the results for 16 ordinates at a later time. We see the the the lumped quantity u is already quite close to the equilibrium value 1 and moreover the solution appears to be very smooth (Note the scaling in vertical direction in Fig. 3).

References

1. Heaslet, M., Baldwin, B.: Predictions of the structure of radiation-resisted shock waves. Phys. Fluids **6**, 781–791 (1963)
2. Kawashima, S., Nikkuni, Y., Nishibata, S.: The initial value problem for hyperbolic-elliptic coupled systems and applications to radiation hydrodynamics. In: Freistühler, H.(ed.) Analysis of Systems of Conservation Laws. Chapman&Hall, 87-127 (1998)
3. Lin, C., Coulombel, J.-F., Goudon, T.: Shock profiles for non-equilibrium radiating gases. Phys. D **218**, no. 1, 83–94 (2006).
4. Mihalas, D., Mihalas, B.: Foundations of Radiation Hydrodynamics. Oxford University Press, New York (1984).
5. Pomraning, G.C.: The Equations of Radiation Hydrodynamics. Pergamon Press, Oxford, New York (1973).
6. Rohde, C.: Entropy solutions for weakly coupled hyperbolic systems in several space dimensions. Z. Angew. Math. Phys. **49**, No. 3, 470-499 (1998).
7. Yong, W.-A.: Entropy and global existence for hyperbolic balance laws. Arch. Rat. Mech. Anal. **172** (2004), 247–266.
8. Rohde,C., Yong, W.-A.: The Nonrelativistic Limit in Radiation Hydrodynamics. I. Weak Entropy Solutions for a Model Problem. Preprint, 2005.
9. Rohde, C., Yong, W.-A.: Shock structure for radiation hydrodynamics. In preparation.

Erschienene Preprints ab Nummer 2006/001

Komplette Liste: `http://preprints.ians.uni-stuttgart.de`

2006/001 *Klimke, A.:* Sparse Grid Interpolation Toolbox - User's Guide

2006/002 *Klimke, A., Wohlmuth, B.:* Constructing Dimension-Adaptive Sparse Grid Interpolants using Parallel Function Evaluations

2006/003 *Hartmann, S., Brunssen, S., Ramm, E., Wohlmuth, B.:* Application of a primal-dual active set strategy for unilateral non-linear dynamic contact problems of thin-walled structures

2006/004 *Sändig, A.-M.:* Partielle Differentialgleichungen Vorlesung im Wintersemester 2005/2006

2006/005 *Nicaise, S., Witowski, K., Wohlmuth, B.:* An a posteriori error estimator for the Lame equation based on H(div)-conforming stress approximations

2006/006 *Geis, W. (ed.), Sändig, A.-M. (ed.):* Second International Workshop - Direct and Inverse Problems in Piezoelectricity,Hirschegg (Kleinwalsertal), Austria, July 16-19, 2006

2006/007 *Klimke, A.:* Efficient Construction of Hierarchical Polynomial Sparse Grid Interpolants using the Fast Discrete Cosine Transform

2006/008 *Buchukuri, T., Chkadua, O., Natroshvili, D.:* Mathematical modelling and analysis of interaction problems for metallic-piezoelectric composite structures with regard to thermal stresses

2006/009 *Merkle, C., Rohde, C.:* The Sharp-Interface Approach for Fluids with Phase Change: Riemann Problems and Ghost Fluid Techniques

2006/010 *Sändig, A.-M.:* Regularity results for linear elliptic boundary value problems in polygons. Lectures at the Charles University Prague, Oct.05

2006/011 *Flemisch, B., Kurz, S., Wohlmuth, B.:* A Framework for Maxwell's Equations in Non-Inertial Frames Based on Differential Forms

2006/012 *Sändig, A.-M.:* Nichtlineare Funktionalanalysis mit Anwendungen auf partielle Differentialgleichungen. Vorlesung im Sommersemester 2006

2006/013 *Rohde, C., Tiemann, N., Wen-An Yong:* Weak and Classical Solutions for a Model Problem in Radiation Hydrodynamics